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A new criterion for end-conduction effects in hot-wire anemometry

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Abstract
The effect of end conduction on constant temperature hot-wire anemometry was studied. A new parameter, $\Gamma = (\ell/d)\sqrt{4a(k_f/k)Nu}$, is proposed to describe the significance of end conduction more comprehensively than the commonly used length-to-diameter ratio ℓ/d, in that it allows for material property variations, resistance ratio and Reynolds number effects. Numerical and experimental data are used to show that Γ improves the correlation of the attenuation of measured turbulence fluctuations, and it is found that $\Gamma > 14$ is required to avoid such effects.

Keywords: hot wire, anemometer, end-conduction effects, length-to-diameter ratio

1. Introduction

The basic principle of hot-wire anemometry is that the convective heat transfer from the thin wire filament is related to the Joule heating of the filament by the electric current that passes through the wire. In the constant temperature mode of operation, the flow velocity over the wire can be found by relating it to the voltage across the wire by calibrating the response in a known velocity field. In a finite-length probe, however, a portion of the Joule heating is conducted to the wire support or stubs, and this end-conduction effect attenuates the measured turbulence fluctuations, which leads to undesirable errors in the data. These end-conduction effects must be minimized to acquire accurate data.

The subject of end-conduction effects has therefore received considerable attention in the past, notably by Corrsin (1963), Perry *et al* (1979), Freymuth (1979) and Ligrani and Bradshaw (1987), and more recently by Li *et al* (2004), Li (2004) and Hutchins *et al* (2009). The problem has gained additional urgency in light of the recent work in wall-bounded flows at high Reynolds numbers where the effects of spatial filtering, due to the finite sensor length, have been shown to be the central issue (Hutchins *et al* 2009, Chin *et al* 2009, Smits *et al* 2011, Monkewitz *et al* 2010). To mitigate the effects of spatial filtering, there is a strong motivation to use short probe lengths, which in turn can exacerbate end-conduction effects. The commonly accepted criterion for negligible attenuation of the turbulence measurements due to end conduction, first proposed by Ligrani and Bradshaw (1987), is that the wire aspect ratio ℓ/d exceeds 200, where ℓ is the wire length and d is its diameter. In order to satisfy this criterion, reducing the length of the wire necessitates reducing the diameter of the wire. With conventional manufacturing techniques (such as etching Wollaston wire), the minimum wire diameter is typically about 1 μm before the wire becomes too fragile or becomes increasingly prone to drift, making a reliable calibration impossible. Hence, the minimum achievable wire length is limited to about 0.2 mm using conventional techniques.

Previous literature, however, indicates that the ℓ/d ratio is not the only parameter to describe end-conduction effects, and that the wire Reynolds number and the thermal conductivity of the wire are also important factors. For example, Betchov (1948, 1949) showed that the resistance distribution along the length of the hot-wire filament is described by

$$RI^2 = \pi dh(T - T_0) - \frac{k_w \pi d^2}{4}\frac{d^2T}{dx^2} \tag{1}$$

where $R(x)$ is the resistance per unit length at the wire temperature $T(x)$, x is the coordinate along the wire (see figure 1), I is the current through the wire, k_w is the thermal conductivity of the wire, and h is the convective heat transfer coefficient. The subscript 0 indicates that the parameter is evaluated at the ambient temperature. If the stubs are perfect heat sinks, and if the resistance of the wire is a linear function of temperature, so that

$$R = R_0(1 + \alpha\Delta T), \tag{2}$$

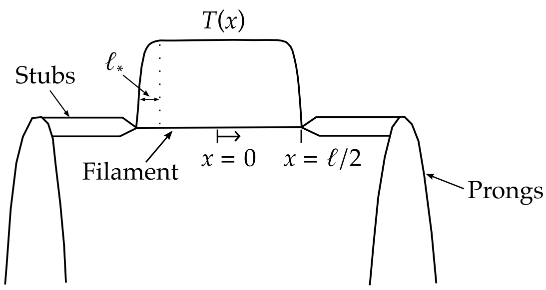

Figure 1. A schematic of a typical hot-wire sensing element with prongs, stubs and an ideal temperature profile.

where α is the thermal coefficient of resistance of the hot-wire material, and ΔT is the temperature difference between the wire and the ambient flow, then the resistance distribution over the wire is given by

$$R(x) = \frac{AR_0}{A - I^2}\left(1 - \frac{I^2}{A}\frac{\cosh(x/\ell_*)}{\cosh(\ell/\ell_*)}\right), \qquad (3)$$

where

$$A = \frac{hd\pi}{R_0\alpha} \quad \text{and} \quad \ell_* = \sqrt{\frac{k_w\pi d^2}{4\alpha(A - I^2)R_0}}. \qquad (4)$$

The parameter ℓ_* is Betchov's 'cold' length, which is a measure of the length of the wire affected by end-conduction effects. Ideally, this cold length needs to be much smaller than the total wire length. Increasing the ℓ/d ratio will decrease ℓ_*/ℓ, but it is clear that it is not the only parameter that will do so.

Freymuth (1979) introduced instead the parameter σ as a better description of end-conduction effects, where σ is the ratio of heat conducted to the total heat transfer. That is,

$$\sigma = \frac{K}{\phi + K}, \qquad (5)$$

where ϕ is the heat convected away from the wire and K is the heat conducted to the stubs. Freymuth found that for the effects of end conduction to be negligible $\sigma < 0.05$. In order to estimate this ratio, Freymuth used the temperature distribution derived by Corrsin (1963) to find the wire Biot-number $h\ell/k_w$, and assumed that the overheat ratio $\overline{R}/R_0 - 1 \ll 1$, where $\overline{R}$ is the average resistance per unit length at the operating condition. This approximation is highly restrictive since in most applications the overheat ratio is actually close to 1 so that the heat conducted to the stubs will depend on the overheat ratio, as shown by Li (2004).

In a complementary work, Li *et al* (2004) numerically solved the governing equation for a hot-wire connected to finite-sized stubs to find the temperature distribution over the wire filament and in the stubs. Their approach is general and allows for heat generation and convection from the stubs and the filament. Different wire materials and flow conditions were simulated and it was shown that the degree of end conduction was a function of the Reynolds number since the higher Reynolds number increases the convective heat transfer as well as the properties of the wire material. It was concluded that for $\sigma < 0.07$ end-conduction effects would be negligible

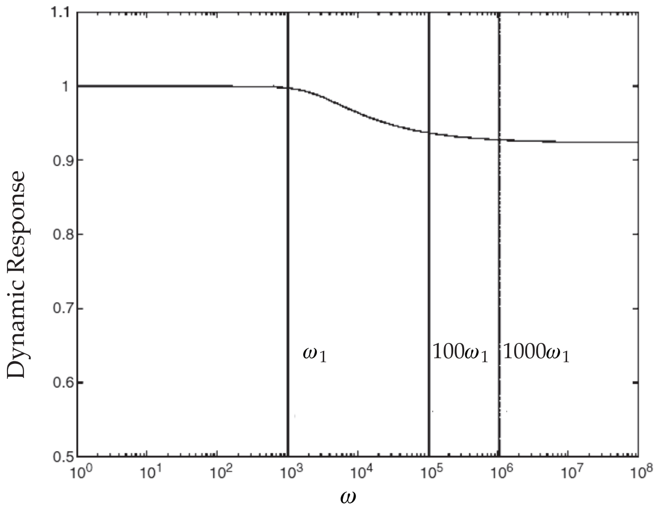

Figure 2. The Bode plot for a hot-wire suffering from end-conduction effects. The attenuation which is seen for frequencies above ω_1 is due to end-conduction effects. Figure taken from Li (2004).

under all circumstances, and that the traditional $\ell/d > 200$ criterion is only valid for platinum wires at typical velocities and at atmospheric pressure. At higher Reynolds numbers, or with a material with lower thermal conductivity, ℓ/d can be considerably smaller while avoiding any significant effect due to end conduction.

Li (2004) related the effects of end conduction to the attenuation of the velocity fluctuation signal for different wire materials and flow conditions. Figure 2 displays the resulting Bode plot for a hot-wire affected by end conduction. Li showed that the level of the attenuation at high frequencies depends on the wire aspect ratio, and introduced an attenuation factor, ε, equal to the ratio of the high-frequency gain to the steady-state gain. The cross-over frequency, ω_1, is given approximately by $40k_w/(\rho_w C_w \ell^2)$, where ρ_w and C_w are the density and heat capacity of the wire material, respectively.

Li's analysis indicates two possibilities for minimizing end-conduction effects: increase ω_1 so that it is higher than any frequency in the flow, or make ε close to unity. The first solution is not possible for most cases, although recent developments in manufacturing nano-scaled hot-wire probes (Bailey *et al* 2010) may make this regime accessible. Here we focus on the second approach, namely to make ε close to unity, and show how the high-frequency gain can be described in terms of a single generalized parameter, Γ, so that when $\Gamma > 14$ end-conduction effects will be negligible. This criterion is intended to replace the less general criterion of $\ell/d > 200$ proposed by Ligrani and Bradshaw (1987).

2. Theory

Here we search for a parameter that describes the behavior of σ rather than requiring the exact solution of the governing equations to find K and ϕ. Consider the case where $K \ll \phi$, or, equivalently, where $\ell^*/\ell \ll 1$, as is the usual case. We note that $1/\sigma \approx \phi/K$, and that the heat lost to the stubs K

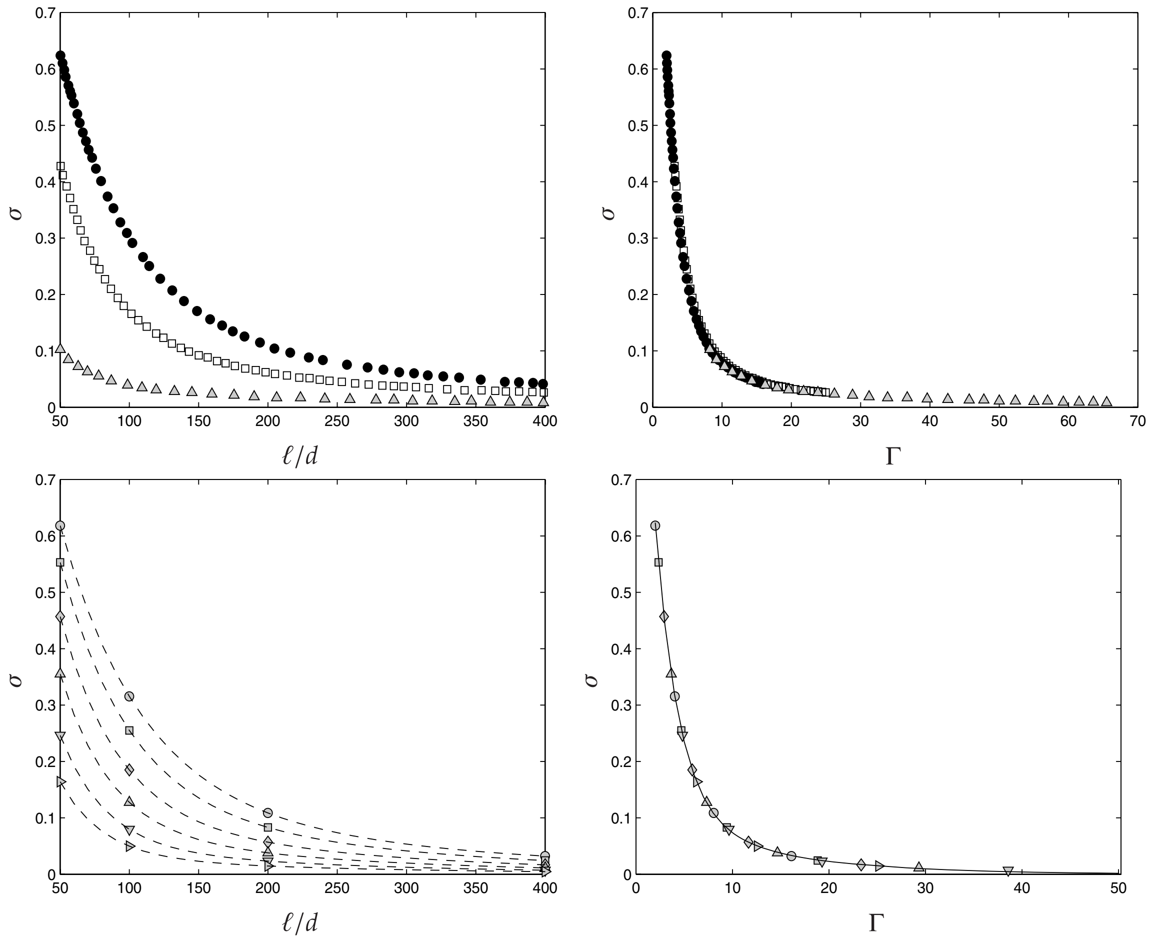

Figure 3. Ratio of end conduction to total heat transfer. Top figures: results from Li *et al* (2004) for three different hot wires (•, tungsten; □, platinum; △, titanium/nickel) operating at $Re_w = 3.2$ and $a = 1.82$. Bottom figures: new simulations following Li *et al* (2004) with different Reynolds numbers (○, $Re_w = 0.3$; □, $Re_w = 1$; ◇, $Re_w = 3$; △, $Re_w = 10$; ▽, $Re_w = 33$; ▷, $Re_w = 100$) for a platinum wire with $a = 1.82$. Left: ratio shown as a function of ℓ/d, indicating the effect of different wire material. Right: ratio shown as a function of Γ, indicating the high level of collapse.

can be found by using equation (3) together with the Fourier heat conduction law. Hence, $K \approx \ell_* R_0 I^2$. At this level of approximation, the convective heat loss is balanced by the Joule heating, so that $\phi \approx \ell \overline{R} I^2$, and

$$\frac{\phi}{K} \approx \frac{\ell \overline{R} I^2}{\ell_* R_0 I^2} = \frac{\ell a}{\ell_*} = \Gamma \qquad (6)$$

where $a = \overline{R}/R_0$ is the resistance ratio and Γ is the desired non-dimensional parameter describing the end-conduction effects. To find an estimate for ℓ^*, we need to find the current through the wire (see equation (4)). We again assume that the convective heat loss is balanced by the Joule heating, so that $I \approx \sqrt{h\pi d \Delta T/\overline{R}}$. Hence,

$$\Gamma = \frac{\ell}{d}\sqrt{4a\left(\frac{k_f}{k}\right)Nu}. \qquad (7)$$

Here k_f is the thermal conductivity of the fluid evaluated at the film temperature. The Nusselt number can conveniently be calculated using a correlation such as that given by Fand (1965):

$$Nu = (0.35 + 0.56 Re_w^{0.52}) Pr^{0.3}, \qquad (8)$$

where Re_w is the Reynolds number based on the diameter of the wire. Note that Re_w is based on the local mean velocity, so that it will depend on the position in a shear flow.

We now propose Γ as a new measure of end-conduction effects. This new parameter extends the traditional ℓ/d criterion to include the effects of the resistance ratio, Nusselt number, and the thermal conductivities of the wire and the fluid. Based on the findings by Li *et al* (2004), we expect that $\Gamma > 14.3$ for end-conduction effects to be negligible (since $\Gamma \approx 1/\sigma$ when end conduction is small).

3. Verification on experiment and numerical data

To verify the universality of Γ, a number of comparisons with previous works as well as new experiments and simulations were made.

First, σ, the ratio of end conduction to the total heat transfer calculated by Li *et al* (2004), is shown in figure 3. In the same figure, new simulations are shown for different values of Re_w. These new cases were simulated following the procedure outlined in Li *et al* (2004), using the same

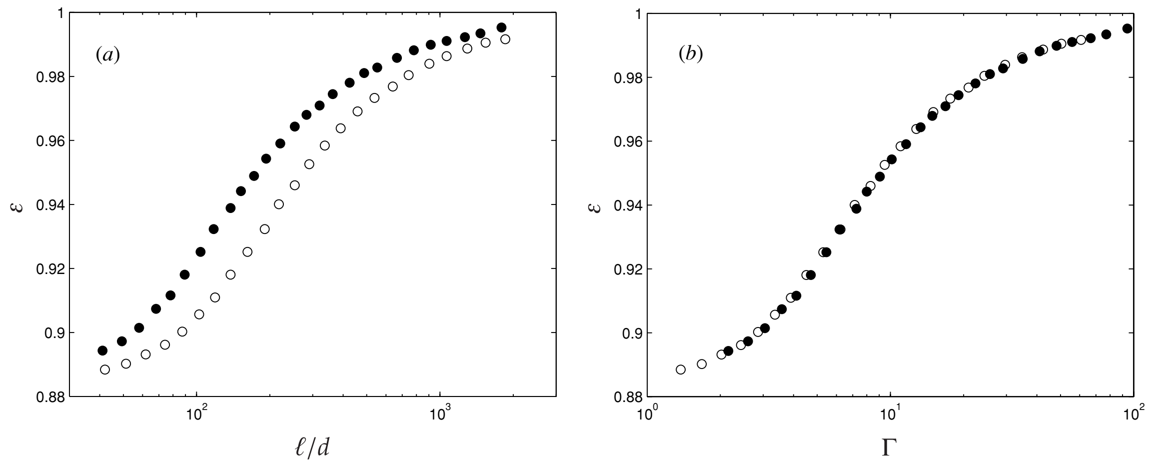

Figure 4. Numerically simulated attenuation to hot-wire data from Li (2004) at $Re_w = 3.2$ for: • platinum wire and ○ tungsten wire, which show the attenuation, ε, as a function of (*a*) ℓ/d and (*b*) Γ.

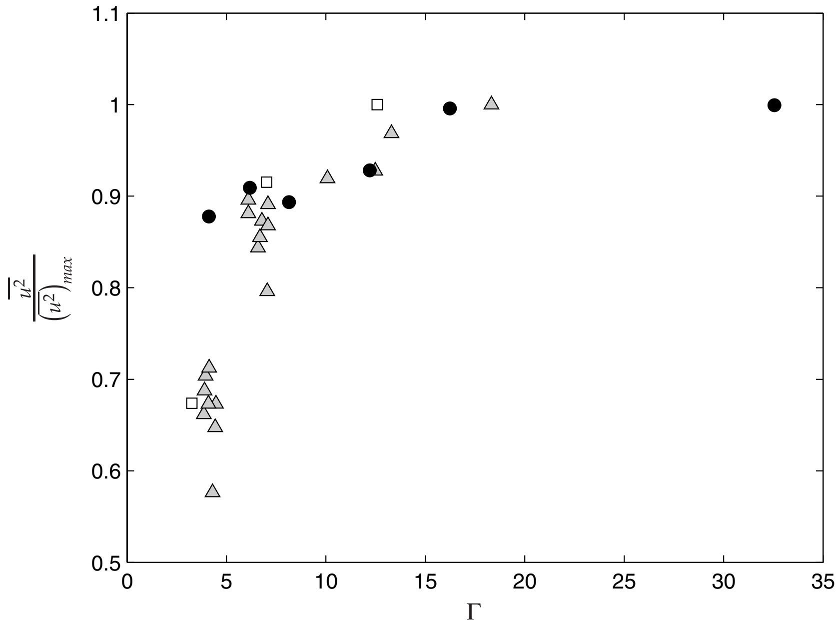

Figure 5. Variances of the streamwise velocity component acquired with different wire aspect ratios at the location of its maximum, normalized by the variance measured with the longest wire in each study. △, data from Ligrani and Bradshaw (1987); □, data from Hutchins *et al* (2009); •, new data from grid turbulence, plotted as functions of Γ, as derived in this paper.

boundary conditions and allowing the stubs to heat up. It is clear that the parameter Γ yields an excellent collapse of the data for different wire materials and for different Reynolds numbers, in contrast to the use of the conventional parameter ℓ/d.

Second, ε, the ratio of the high-frequency gain to the steady-state gain calculated by Li (2004), is shown in figure 4. Again the collapse is excellent when scaled by Γ.

Third, experimental data from Ligrani and Bradshaw (1987) and Hutchins *et al* (2009) were examined. Both data sets were taken in zero-pressure gradient flat plate boundary layers. Hutchins *et al*'s data were taken at $Re_\tau = 14\,000$ where the ℓ/d was varied from 39, 100 and 200. Here $Re_\tau = Ru_\tau/\nu$, where $u_\tau = \sqrt{\tau_w/\rho}$, τ_w is the wall shear stress, and ρ and ν are the fluid density and kinematic viscosity, respectively. To ensure that the data were not corrupted by spatial filtering, only data with constant $\ell^+ = 22$ were considered, where $\ell^+ = \ell u_\tau/\nu$. The data from Ligrani and Bradshaw (1987) were taken at significantly lower Reynolds numbers ($Re_\tau \approxeq 1000$), with $69 < \ell/d < 372$. To minimize spatial filtering only data with $\ell^+ < 10$ are considered. The key parameters pertaining to these investigations are summarized in table 1.

In figure 5 we compare the variance of the streamwise turbulent velocity at the point of maximum fluctuations (which occurs at a wall distance y where $y^+ \approx 15$) for different wire lengths, normalized by the variance acquired with the longest

Table 1. The key parameters for the two different experimental data sets considered in this paper.

	d (μm)	ℓ (mm)	Re_w	k_w (W m^{-1} K^{-1})	ℓ^+
Hutchins *et al* (2009)	12.7, 5, 2.5	0.5	17, 6.7, 4.3	69	22
Ligrani and Bradshaw (1987)	0.625, 1.25, 5	0.05	0.3–2.5	38	1–10

wire for the two experimental data sets considered, plotted as functions of Γ. To further validate the universality of Γ, new data from a grid turbulence experiment are also plotted in figure 5. The grid turbulence was generated in a 0.61 m by 0.91 m closed circuit wind tunnel using a 25.4 mm square mesh grid with a solidity of 23% (for more details on the experimental setup, see Bailey *et al* 2010). Experiments were performed 65 mesh lengths downstream of the grid at 10 m s^{-1} which corresponds to a Reynolds number based on the Taylor micro-scale, $Re_\lambda = 32$. Wollaston wires (90% Pt–10% Rh) with a diameter of 5 μm were used, operated with $a = 1.5$. The wire length was varied between 0.25 and 2 mm corresponding to a range in ℓ/η of 0.7 and 5.3 for the different wires used, where η is the Kolmogorov length scale, which was estimated by measuring the rate of decay of the variance and assuming homogeneity and isotropy. In order to separate the end-conduction effects from spatial filtering, the data were corrected using the spatial filtering correction developed by Bailey *et al* (2010). The correction was always less than 5% of the measured variance. An excellent collapse is observed to within experimental error, and we see that $\Gamma > 14$ is a sufficient criterion to avoid end-conduction effects. Some of the scatter that persists in the data may be attributed to the fact that one is confusing spatial filtering effects with end-conduction effects since the data by Ligrani and Bradshaw (1987) were acquired at slightly different values of ℓ^+.

4. Conclusions

A new parameter, $\Gamma = (\ell/d)\sqrt{4ak_f Nu/k_w}$, derived using the analysis of Betchov (1948) and the approximation that the conduction to the stubs is small compared to the Joule heating was found to describe accurately end-conduction effects in hot-wire anemometry. We therefore recommend the Γ criterion to replace the conventional ℓ/d criterion to describe end-conduction effects, and propose that $\Gamma > 14$ is necessary to avoid any attenuation in the turbulent fluctuations. The parameter Γ includes the conventional end-conduction parameter ℓ/d, but it also takes into account the effects due to hot-wire material, overheat ratio and Reynolds number. We note that wires can now be designed to avoid end-conduction effects even when not satisfying the conventional $\ell/d > 200$ criterion.

Acknowledgments

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References

Bailey S C C, Kunkel G J, Hultmark M, Vallikivi M, Hill J P, Meyer K A, Tsay C, Arnold C B and Smits A J 2010 Turbulence measurements using a nanoscale thermal anemometry probe *J. Fluid Mech.* **663** 160–79

Betchov R 1948 L'influence de la conduction thermique sur les anemomètres à fils chauds *Verh. K. Akad. Wet.* **51** 721–30

Betchov R 1949 Theorie non-linéaire de l'anemomètre à fil chaud *Verh. K. Akad. Wet.* **52** 195–207

Bruun H H 1995 *Hot-Wire Anemometry* (Oxford: Oxford University Press)

Chin C C, Hutchins N, Ooi A S H and Marusic I 2009 Use of direct numerical simulation (DNS) data to investigate spatial resolution issues in measurements of wall-bounded turbulence *Meas. Sci. Technol.* **20** 115401

Corrsin S 1963 *Turbulence: Experimental Methods* (*Handbuch der Physik* vol 8) (Berlin: Springer)

Fand R M 1965 Heat transfer by forced convection from a cylinder to water in cross flow *Int. J. Heat Mass Transfer* **8** 995

Freymuth P 2002 Engineering estimation of heat conduction loss in constant temperature thermal sensors *TSI Q.* **5** 3–8

Hutchins N, Nickels T B, Marusic I and Chong M S 2009 Hot-wire spatial resolution issues in wall-bounded turbulence *J. Fluid Mech.* **635** 103–36

Li J D 2004 Dynamic response of constant temperature hot-wire system in turbulence velocity measurements *Meas. Sci. Technol.* **15** 1835–47

Li J D, McKeon B J, Jiang W, Morrison J F and Smits A J 2004 The response of hot wires in high Reynolds-number turbulent pipe flow *Meas. Sci. Technol.* **15** 789–98

Ligrani P M and Bradshaw P 1987 Spatial resolution and measurement of turbulence in the viscous sublayer *Exp. Fluids* **5** 407–17

Monkewitz P A, Duncan R D and Nagib H M 2010 Correcting hot-wire measurements of stream-wise turbulence intensity in boundary layers *Phys. Fluids* **22** 091701

Perry A E 1982 *Hot-Wire Anemometry* (Oxford: Clarendon)

Perry A E, Smits A J and Chong M S 1979 The effects of certain low frequency phenomena on the calibration of hot wires *J. Fluid Mech.* **90** 415–31

Smits A J, Monty J, Hultmark M, Bailey S C C, Hutchins N and Marusic I 2011 Spatial resolution correction for wall-bounded turbulence measurements *J. Fluid Mech.* at press