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Elastic filament velocimetry (EFV)

M K Fu, Y Fan, C P Byers, T-H Chen, C B Arnold and M Hultmark

Abstract

A novel method for velocity measurements in both gaseous and liquid flows is presented. The sensing element is comprised of a free-standing electrically conductive nanoscale ribbon suspended between silicon supports. Due to its minuscule size, the nanoribbon deflects in flow due to viscously dominated fluid forcing inducing an axial strain of the sensing element. The strain leads to a resistance change, which is measurable through a simple Wheatstone bridge circuit and can be related to the flow velocity through semi-analytic analysis. Two methods of characterization are employed to validate the sensor functionality. First, confocal microscopy was used to validate physical models and assumptions through imaging of the nanoribbon deformation under different fluid loads. Second, the resistance measurements of various nanoribbons under different flow conditions exhibited sensitivity to fluid flow consistent with lower order model predictions.

Keywords: sensor, flow measurements, MEMS, velocity measurements

1. Introduction

A strain based sensor is developed to measure the velocity of a passing fluid through the use of a free-standing, electrically conductive nanoscale ribbon. Drag from the passing flow deflects the nanoribbon and induces an axial strain within the material. The small dimensions of the sensing element result in viscously dominated drag force, enabling sensitivity to both liquids and gases. The combination of the elongation, thinning, and piezoresistive effects relating to the axial strain generates a change in electrical resistance across the nanoribbon, i.e., a strain gauge effect. Because the fluid velocity can be uniquely related to the fluid drag on the wire, the flow velocity can be directly correlated with the resistance of the wire.

The idea of using calibrated strain gauges to measure fluid velocity is not new [1–3]. Most of these ideas involve utilizing calibrated cantilevers or plates embedded with strain gauges on one or more surfaces. The bending and deflection of the entire member is then calibrated to the fluid velocity. The device described here is far simpler in design and operation, as the calibrated member and strain gauge are one and the same. The construction and form of the sensor is similar to the design of a nanoscale hot-wire, while the operation of the sensor is similar to that of a strain gauge. This hybrid design eliminates the most complicated and expensive aspects of each sensing method, resulting in simpler design and operation.

The novelty of the nanoribbon design lies in the relative geometry and overall size of the sensing element, made possible by advancements in microelectromechanical systems (MEMS) manufacturing techniques. The nanoscale thickness of the free-standing nanoribbon allows it to have a negligible flexural rigidity. Supporting the free-standing nanoribbon at two ends constrains the transverse deflections to be small compared to the span, while allowing it to undergo measurable strains. Furthermore, the small transverse deflection to span ratio ensures that the fluid flow is restricted to a quasi-2D regime, ensuring that spanwise changes along the wire have negligible effects and interaction on the fluid flow. Finally, the overall microscale dimension of the wire profile allows a linear relationship between the fluid forcing on the wire and the flow velocity. The consequences of these geometric properties enable both a high degree of sensitivity and conceptual simplicity that facilitates analytic modeling [4, 5].

The nanoribbon configuration was inspired by the sensor designs of the nano-scale thermal anemometry probe (NSTAP) [6, 7] and its cold-wire counterpart (T-NSTAP) [8]. The T-NSTAP, a platinum nanoribbon with dimensions of span \( L_0 = 200 \, \mu\text{m} \), width \( b = 2 \, \mu\text{m} \), and thickness \( t = 100 \, \text{nm} \), (see figure 2) was deployed for use in a water channel. It
was revealed that the wire exhibited a sensitivity to the flow velocity, in addition to the sensitivity to temperature changes in the channel, when it was mounted with an angle to the incoming flow. Holding the bulk temperature of the flow constant, a calibration of the T-NSTAP voltage to flow velocity was performed against a Pitot tube for a range of velocities. The sensor was then traversed through the boundary layer, and the data was corrected for wall temperature of 0.3 °C higher than the free-stream temperature. Figure 1 shows a convincing agreement between the corrected measurements and classical curve fits for a turbulent boundary layer, with error bars demonstrating the sensitivity to the temperature correction. This basic proof of concept had shown the investigators herein that a strain-based sensor, using a free-standing nanoribbon design pinned at both ends, is indeed practical.

2. Description

2.1. Theory of operation

The novelty of the sensor lies in the simplicity of the form and function. As the free-standing nanoribbon is exposed to fluid flow, it will deform and elongate. The large aspect ratios enables semi-analytic treatment of the fluid loading and elastic deformation, while the nanoscale thickness enables a high degree of sensitivity. To design a nanoribbon with optimal sensitivity for a given velocity range, relationships must be established between axial strain and fluid velocity. Consider the nanoribbon with rectangular cross section illustrated in figure 2. The span, \( L_0 \), is much greater than the width, \( b \), and thickness, \( t \), the latter of which shall be considered the nanoscale dimension. Under a uniformly distributed load, \( q \), the nanoribbon will experience a maximal deflection, \( \delta \), and overall elongation, \( L = L_0 \). Choosing a nanoscale thickness, several orders of magnitude smaller than the other geometric dimensions, enables the wire to exhibit negligible flexural rigidity to loading along that dimension. The result is that the governing force balance is dominated by the internal tension rather than classic Euler–Bernoulli bending.

Consequently, any component of fluid flow aligned with the nanoscale dimension will elastically elongate the wire until the streamwise component of the tensile stress is sufficient to balance the fluid loading. This net elongation can be measured as a resistance change across the length of the nanoribbon. The deformation, and the corresponding elongation, from an external loading can be determined using beam theory [4,5]. The steady-state deflection of the nanoribbon of uniform cross section under loading \( q \) is governed by

\[
E_j \frac{d^4w}{dx^4} - N \frac{d^2w}{dx^2} = q, \tag{1}
\]

where \( E \) is the Young’s modulus of the material, \( I \) is the second moment of area and given as \( I = \frac{bt^3}{12} \) for this configuration, \( w \) is the deflection from the neutral axis, \( N \) is the axial tension in the beam arising from elastic deformation, and the cross sectional area, \( A \), is constant and given by \( A = bt \). Furthermore, the nanoribbon is assumed to have a large radius of curvature, \( R \), where \( R \gg L_0 \). This assumption, the small angle approximation, will be called upon repeatedly in this analysis and its validity for this sensor was evaluated using confocal microscopy. The small angle approximation allows treatment of the local nanoribbon coordinate system as aligned with the global cartesian system in figure 2. This assumption enables significant simplification of the governing dynamics and can be validated with consideration of the particular flow and wire parameters. Specifically, \( N \) in equation (1) can be treated as constant through the span of the wire.

Scaling of this equation in terms of dimensionless parameters and dimensional coefficients reveals the relative importance of the Euler–Bernoulli term to the internal tension. The given parameters in equation (1) can be scaled as \( w = \delta w^* \) and \( x = L_0 x^* \), where starred parameters are dimensionless and order unity and are preceded by an appropriate dimensional scaling constant. Note that though \( \delta \) is defined as the midpoint deflection, the scale relative to the other parameters is not yet known. Assuming that \( N \) is the tension arising solely from the loading induced strain in the nanoribbon (a flat ribbon with no stress when unloaded), \( N \) can be scaled as \( N = EA \delta^2 L_0^2 N^* \). Expressing equation (1) in terms of these dimensionless parameters yields

\[
0 = -\frac{E \delta t^3}{L_0^4} \left( t^2 \frac{d^4w^*}{dx^4} \right) - \left[ N^* \frac{d^2w^*}{dx^2} \right] + q. \tag{2}
\]

Terms inside of the square brackets of equation (2) are comprised solely of dimensionless scaling functions and variables and are therefore order unity, provided that the proper scales were chosen. Each bracketed term is preceded by a grouping of scaling parameters that indicates the relative importance of each dimensionless scaling function to the force balance. When \( \delta \) is such that \( \delta \gg t \), the first bracketed term is negligible to the overall force balance and the scale of the deflection can

![Figure 1. Corrected velocity profile of a boundary layer measured with a 200 μm long platinum nanoribbon, mounted at an angle to the flow, and plotted in inner coordinates. Flow conditions are \( Re = 1218 \), with \( Re = \rho u_\tau \delta \mu / \mu \) and \( u_\tau = \sqrt{\tau_\delta / \rho} \). The approximate fits for Spalding’s law of the wall (---) and Coles’ law of the wake (--) are included for comparison.](image-url)
be found to vary as \( \delta \sim (qL^4(Ebt)^{-1})^{1/6} \) (further details on the scaling can be found in appendix A). It can also be shown that for the small-angle approximation to be valid, the nanoribbon must be in a regime where \( \delta \ll L_0 \). Collectively, these restrictions can be expressed as conditions on the forcing scale, \( q \), where

\[
\frac{Ebt^4}{L_0^4} \ll q \ll \frac{Ebt}{L_0}.
\]

(3)

Together, these criteria specify the minimum design criteria to ensure that the governing equations apply to the specific nanoribbon geometry, though they ignore specific material limitations such as yield strength. As \( N \) is the tension derived from axial elongation, equation (1) can be expressed as

\[
-EA\frac{\delta w}{dx^2} = -\frac{EA}{L_0} - \frac{Ebt}{L_0^2} \int_{-L_0/2}^{L_0/2} \left( 1 + \left( \frac{dw}{dx} \right)^2 \right) \frac{d^2w}{dx^2} = q.
\]

(4)

The small angle approximation can be invoked to simplify the integration through the treatment of \( w \) as a second order Taylor approximation. Using the coordinate system outlined in figure 2, \( w \) can be accurately represented by \( w(x) \approx \delta (1 - 4x^2L_0^2) \). Given that \( \frac{dw}{dx} \ll 1 \), the integrand in equation (4) can also be simplified as a second order Taylor approximation to give

\[
-EA\frac{\delta w}{dx^2} = \left( \frac{EA}{2L_0} - \frac{Ebt}{L_0^2} \int_{-L_0/2}^{L_0/2} \left( \frac{dw}{dx} \right)^2 dx \right) \frac{d^2w}{dx^2} = q.
\]

(5)

Applying the parabolic approximation for the shape of the deflected beam, an explicit solution to this equation can be achieved. A simple integration of a parabolic arc length along the axis of the nanoribbon reveals that the strain \( \varepsilon \) of the nanoribbon can be expressed in terms of the \( L_0 \) and \( \delta \) as

\[
\varepsilon = \frac{8\delta^2}{3L_0^2}.
\]

(6)

Integrating equation (5) presents a relationship between the axial stress induced by the strain and the uniform loading, given by

\[
EA\varepsilon = q\frac{L_0^2}{8\delta}.
\]

(7)

The above equation is consistent with a force balance where the external loading must be balanced by vertical component of the force applied at the pinned ends of the nanoribbon. The midpoint deflection can be solved for explicitly as

\[
\delta = \left( \frac{3qL_0^4}{64EA} \right)^{1/3},
\]

(8)

and the strain as

\[
\varepsilon = \frac{1}{2} \left( \frac{qL_0^2}{\sqrt{3EA} \delta} \right)^{2/3}.
\]

(9)

An important consequence of equations (8) and (9) is that the functional relationship is consistent with the scaling analysis derived from equation (2).

To ensure that the modeling is applicable, \( q \) must be related to the fluid flow to determine if the criteria outlined in equation (3) remains valid. In the small angle approximation, changes and gradients along the length of the wire are considered insignificant compared to the streamwise and transverse gradients of the flow. It is therefore appropriate to treat the fluid mechanics in a quasi-2D manner. This implies that the deflections of the beam have a negligible effect on the fluid gradients and the loading on the beam can be related to the local flow velocity around the nanofilament. Additionally, if \( b \) is sufficiently small so that the Reynolds number based on \( b, \text{Re}_b \equiv \rho U b \mu^{-1} \ll 50 \), then the flow can be considered viscously governed and the drag from the flow can be accurately described using semi-analytic techniques. Here, \( \rho \) is the fluid density, \( \mu \) is the dynamic viscosity of the fluid and \( U \) is the incoming flow velocity. Under these conditions, the local load per unit span, \( q(x) \), from the fluid can be described in a linear fashion as,

\[
q(x) = C_D U(x) \mu
\]

(10)

where \( U(x) \) is a velocity component of the flow aligned with the thickness dimension and \( C_D \) is the coefficient of drag. While there is no analytic solution for low Reynolds’ number flow over a 2D shape, there are several approximations that specify

**Figure 2.** A schematic of the nanoribbon of length \( L \), width \( b \), and thickness \( t \). The geometry is oriented such that flow is in the direction of \( \hat{x} \), causing a distributed load, \( q \), that leads to a center-line deflection of \( \delta \). The platinum nanoribbon used to generate the results in figure 1 had \( L_0 = 200 \mu m, b = 2 \mu m, t = 100 nm \).
The size of the maximum velocity criteria indicates that it is significantly larger than any small angle approximation. Reassuringly, this result indicates that the small angle approximation should be valid for all velocities of interest, for this particular wire configuration. Furthermore, one could rearrange the terms in inequality (12) to establish a criteria for \( t \), given a wire length, material and flow of interest.

With the wire mechanics related to the fluid flow, the classical strain gauge equations can be utilized to relate the strain in the nanoribbon to a measured resistance change. Through a simple first order Taylor series approximation, it can generally be shown that

\[
\frac{\Delta R}{R_0} \approx \frac{1}{R_0} \frac{\partial R}{\partial \varepsilon} \bigg|_{\varepsilon=0} \varepsilon = \left( 1 + 2\nu + \frac{1}{\mu_0} \frac{\partial \mu}{\partial \varepsilon} \right) \varepsilon = G\varepsilon. \tag{13}
\]

where \( R \) is resistance of the wire defined in the classical manner \( R = \rho LA^{-1} \), \( \Delta R \) is the change in resistance from the reference resistance due to strain, \( \nu \) is Poisson’s ratio for the wire material, \( \rho \) is the wire resistivity, and the subscript 0 denotes the specific reference state of known temperature and strain. The first two parenthetical terms incorporate the geometric elongation and narrowing derived from strain, while the third is contribution from piezoresistivity. Each of the terms contained in the parenthesis in equation (13) are material properties and independent of wire geometry. It is common to express these terms collectively as a single material specific gauge factor, GF, which relates the fractional change in resistance to the strain. Collecting the results from equations (11) and (13), a functional relationship between the fluid loading and measured resistance is given by

\[
\frac{\Delta R}{R_0} = \frac{GF}{2} \left( \frac{C_{Dh}UL_0}{\sqrt{3}EA} \right)^{2/3}. \tag{14}
\]

Though the relations used to derive equation (14) neglected the effects of pre-stress or pre-deformation, equation (7) can be modified to include these effects and a more complicated approximation for the strain can be derived (see appendix B). It can be shown that the net effect of both of these effects is to depress the change in strain from the external loading. Furthermore, if the pre-stress is significantly smaller than the fluid loading, then the effects on the mechanics are negligible.

### 2.2. Sensor manufacturing

With the functional relationship between the fluid load and the wire resistance established in equation (14), an optimal wire will have high gauge factor GF, low Young’s modulus \( E \), large span \( L_0 \), and small cross sectional area \( A \). In other words, for a given fluid flow, a long and thin wire will be more sensitive; a carefully chosen material with low \( E \) and high GF will further boost the sensitivity. To ensure the fluid forcing is in the viscously dominated regime, the width of the wire, \( b \), needs to be small, on the order of micrometers.

Platinum (Pt) has a moderate Young’s modulus among metals, but is non-reactive and easy to process using standard MEMS techniques to form thin films. As a noble metal, no oxidation layer will form to complicate the theory or experiments.
Furthermore, the large piezoresistivity of platinum among metals [10] also adds to the GF to make it a preferred metal for nanoribbon manufacturing.

Pipe flow is one of the simplest flows to study and its velocity profile is well studied across a wide range of Reynolds numbers. The sensor is designed to interface directly with the pipe used in this experimental study by having a 4 mm through-hole, the center of which is spanned by the sensing nanoribbon. The length-to-width ratio of the free-standing nanoribbon is limited by the strength of the metal thin film and was kept under 150 to maintain the integrity of the wire during processing. Minimal supporting structures were deployed to hold the nanofilament to minimize blockage effects that could alter the flow in the vicinity of the sensing element.

A compact layout of the elastic filament velocimetry (EFV) chip is shown in figure 3(a), where the sensing wire shown has dimensions of 750 × 6.5 μm. The black circle shows the position of the 4 mm hole and the area colored in red will be deposited with platinum. The EFV chip is designed to be mounted to a larger printed circuit board (PCB) with the same sized through-hole and electrical leads for taking measurements. Even though platinum is the preferred metal for laboratory fabrication of the EFV, one main disadvantage is that platinum has a moderate temperature coefficient of resistance (≈ 4 × 10⁻³ K⁻¹), meaning its resistance can easily be influenced by changes in the ambient temperature. In fact, because platinum has a near-constant temperature coefficient for a large range of temperature, this property has been commonly used for sensors such as hot- and cold-wires. To decouple the contributions from velocity and temperature towards the resulting resistance changes, a second platinum is etched through, the silicon will release energy and rupture around the first point of opening (figure 4(b)). Since the platinum nanoribbon is only 150 nm in thickness, a rupture would most likely cause the wire to break and render the sensing element unusable. Therefore, obstructive ‘islands’ are added to create barriers and divide the hole into smaller areas. In each divided area, the center tends to etch slightly faster, making it the first point to etch through.

Different lattice spacings and elevated temperature during nitride deposition are responsible for a residual stress at the silicon wafer and silicon nitride interface. When the wafer is etched through, the silicon will release energy and rupture around the first point of opening (figure 4(b)). Since the platinum nanoribbon is only 150 nm in thickness, a rupture would most likely cause the wire to break and render the sensing element unusable. Therefore, obstructive ‘islands’ are placed in an asymmetric fashion, as seen in figure 3(b), promoting the rupture to occur away from the wire. The introduction of asymmetric ‘islands’ during through-hole etching proved to significantly increase the yield of the sensor manufacturing process (figure 4(c)).

Once the chips are released from the wafer, another silicon dry etch with sulfur hexafluoride is performed to clean up excess silicon in the through-hole and remove the PECVD.
silicon nitride. The chip is then mounted to a PCB with conductive epoxy as the bonding agent for good electrical connections (figure 5).

### 2.3. Experimental setup

The sensor was evaluated using the experimental setup shown in figure 6, which was configured to use either water, air or nitrogen as the working fluid. Regulated fluid flow was directed through a section of calibrated smooth pipe. Bulk flow velocity was determined by measuring the pressure drop over a length, $l$, where the flow was fully developed. Flow exiting the system would pass over the wire, inducing a strain and resistance change. The fractional resistance changes were measured by integrating the nanoribbon as part of a Wheatstone bridge. The voltage across the bridge was directly related to the resistance change in the wire through the known values of the bridge circuit components. Each of these components in the bridge were carefully chosen to minimize the resistive heating in the wire, while ensuring a measurable level of sensitivity to flow loading. The circuitry was designed to limit the current in the Wheatstone bridge to 67 $\mu$A, which means less than 1% of measured resistance change is due to Joule heating if the wire is under water. However, Joule heating will contribute a larger change in wire resistance when in air, especially at lower velocities. In experiments conducted with confocal microscopy, the objective lens was placed above the outlet of the system so that the nanoribbon deflections were perpendicular to the focal plane.

A thermocouple was placed at the outlet of the flow, less than 1 cm downstream of the EFV chip. The introduction of the thermocouple was necessary, as the second wire was found to also deflect when exposed to flow.

### 3. Results

#### 3.1. Deflection measurements

Laser scanning confocal imaging of a $750 \mu$m $\times$ $6.5 \mu$m $\times$ $150$nm sensor, shown in figure 7, was performed while the nanoribbon was exposed to flow to further investigate and validate the modeling in section 2.1. Two-dimensional area maps of the nanoribbon elevation (figure 7), were recorded for several different flow velocities of nitrogen. From each area map, a centered, linear height profile of the nanoribbon was extracted, as seen in figure 8. Numerical integration was performed along the length of each profile to find the elongation due to the flow, shown as figure 9.

The profile of the unloaded nanoribbon, the bottom-most profile in figure 8, indicates a pre-existing, asymmetric deformation. As the loading increases, so does the net deflection of the nanoribbon, and the shape of the deflection away from the boundary resembles the expected parabolic curve. The remaining deviation from a symmetric, parabolic shape, particularly near the edges, can be best explained by the presence of two triangular platinum supports that are partially freestanding as well. The relative size and shape of the deflections validate that the parabolic shape and small angle approximations are appropriate for the wire mechanics. Furthermore, the relative size of the deflection compared to the thickness implies that the force balance is dominated by the elastic elongation rather than flexural rigidity.

The strain measurements resulting from the microscopy are compared to the theoretical prediction for $\varepsilon$ from equation (11) in figure 9. The bulk value for the Young’s modulus of platinum is taken as a reasonable approximation for $E$ of the nanoribbon [11]. Platinum’s small coefficient of linear thermal expansion further ensures the validity of our theoretical strain predictions, as the small temperature fluctuations in the flow will not have a measurable impact on the
Elongation. Given that $C_D$ is known to increase with Reynolds number, a functional relationship between $Re$ and $C_D$ is necessary to determine the fluid forcing. To approximate this trend, an empirical relation of $C_D$ and $Re$ for a cylinder of diameter $b$ is applied [12].

$$C_D \approx 0.59Re_b^{0.11} + 3.4Re_b^{0.5} - \frac{0.0002Re_b^2}{1 + 3.64 \times 10^{-7}Re_b^2}$$

Though there is no expectation that $C_D$ of a cylinder should match that of the nanoribbon shape, this estimate should capture the overall magnitude and trend of the drag over the span of Reynolds numbers considered in this study.

Despite the uncertainty in $C_D$, there is a reasonable agreement between the slopes predicted by the model and the experimental results. The results deviate from the simplest form of the model, where pre-stress and pre-deflection are neglected, but adding the pre-deflection at zero velocity, $\sigma_0\tau$, shows an example of the strain that the wire could be expected to take while exposed to flow.

### 3.2. Resistance measurements

With the success of the confocal imaging in confirming the scaling theory and the operational principle of the sensor, the resistance changes in the wire were then tested. Using the strains from the confocal imagery, the gauge factor, $GF$, can be estimated to relate the strain in the wire to the measured change in resistance. The observed estimate of $GF = 2.4$ deviates from the expected value of $GF = 6.1$ for platinum as the piezoresistive effect was likely reduced by the particular manufacturing process [13]. Figure 10 shows the estimated strain from both the air and water tests plotted against the non-dimensional fluid forcing, $\frac{3C_D\mu UL/AE}$. The sensors were able to accurately capture the modeled wire strain from the measurement of $\frac{RR}{\Delta}$. Quiescent temperature calibrations for each wire were performed before data collection in order to remove any temperature dependencies of the resistance measurements. To monitor the temperature of the nanoribbon during the flow experiments, a thermocouple was placed in the flow no more than 1 cm downstream of the wire. Using these temperature measurements, the resistance change of the wire due to a temperature change in the flow was subtracted out, and the remaining $\frac{RR}{\Delta}$ was then converted to a strain using the estimated GF value from the confocal data.

The use of a thermocouple was necessary due to the second platinum wire (which was placed near the edge of the flow field, as seen in figure 3) also deflecting when exposed.
to flow. The second wire was originally designed to register temperature measurements without deflecting due to the fluid forcing, thus allowing the two wires to measure both velocity and temperature simultaneously. Instead, since these two variables could not be decoupled with this current sensor and circuit design, the thermocouple was employed to record the flow temperature. In future designs these effects can be decoupled and automatically compensated by incorporating two wires with different geometries in the same Wheatstone bridge.

Figure 9. Strain measurements of a $750 \times 6.5 \, \mu m$ wire from the confocal microscope imaging data. The diamonds display the experimental data. The solid line shows the theoretical prediction from equation (11). The dashed line (−) shows the predictions for finite pre-deflection corresponding to $\sigma_0 = 8 \times 10^{-4}$ from appendix B.

Figure 10. Comparison of the scaling theory with experimental runs in both air and water. Resistance change calculated from voltage measurements while under flow. $C_D$ approximated with that of a cylinder with diameter $b$ for this flow regime and GF approximated at 2.4 for platinum. Loading is estimated using $U_c$, the centerline velocity of the pipe. Symbols: $\Box 750 \times 6.5 \, \mu m$, $\bigcirc 750 \times 6.5 \, \mu m$ (from Confocal Imaging), $\bigtriangleup 375 \times 6.5 \, \mu m$, $\bigtriangledown 375 \times 2.5 \, \mu m$. Dark gray symbols for measurements in water, light gray for measurements in air. The solid line shows the theoretical prediction from equation (11). The dashed line (−) shows the predictions for finite pre-deflection corresponding to $\sigma_0 = 8 \times 10^{-4}$ from appendix B.
In figure 11 the results of only the 750 × 6.5 µm sensor is compared to the theoretical predictions for the velocity measurement. The additional theory line includes the factor $\sigma^+ \varepsilon$ to account for any pre-deflection in the wire due to the manufacturing process (see appendix B). The exact magnitude of the pre-deflection in the experimental setups are not known, but an example value is illustrated in the figure.

The uncertainty in wire temperature are represented by the error bars of figure 11. The estimated errors increase with decreasing fluid forcing. This is especially important in the water tests where heat transfer from the water to the pipe in the system becomes more pronounced, causing the temperature of the fluid to fall (due to the water reservoir have a slightly higher temperature than the pipe system). As a consequence, an artificially higher resistance will be recorded. This temperature uncertainty is approximately accounted for by the thermocouple, but the finite distance between the thermocouple and the EFV result in a different temperature measured than that of the wire. The faster velocities result in a more accurate temperature measurement, since there is less temperature drop between the EFV and the thermocouple. Additional uncertainty lies in the temperature calibration of the EFV itself. A shift of 0.005 °C in the calibration offset results in a change in $\varepsilon$ by up to 0.0001, based on the assumed GF. This sensitivity to temperature is a direct consequence of the moderate temperature coefficient of resistance associated with platinum.

For the data taken in air, the velocities are much higher and the thermocouple can be expected to accurately capture changes in the wire temperature. However, the uncertainty due to a calibration offset remains which results in error bars of equal size across all measurements, in contrast to the increasing error with decreasing velocity in water.

The large magnitude of the error bars indicate that a precise measurement of the EFV temperature is paramount to improving the accuracy of the sensor. A second wire exposed to the flow but fixed to prevent any deflection would allow a very accurate temperature measurement in close proximity to the EFV, enabling a precise temperature calibration of the sensor itself.

4. Conclusion

Utilization of a nanoribbon as a strain-based velocity sensor has proven to be a practical methodology, with simple circuitry and calibration techniques leading to quick and inexpensive data collection. The combination of form and function enables characterization of the sensor design with a simple, low order model, derived from classical beam theory. Measurements from both confocal microscopy and electrical resistance were shown to be consistent and comparable, despite uncertainties in temperature and modeling parameters.

As was demonstrated in the results, the sensitivity of the resistance measurements to temperature is a known problem with strain gauges. Should a nanoribbon be exposed to a nonisothermal flow, it will experience resistance changes due to both the fluid forcing and thermal variations. Reduction of this temperature uncertainty will significantly improve the velocity resolution of the sensor and robustness of the sensor measurements. Several methods and designs exist to mitigate the temperature dependence of typical strain gauges, such as using temperature insensitive alloys like Constantan. Utilizing multiple wires with different sensitivities to loading and temperature (e.g. different stiffness, geometry, GF, etc) will enable simultaneous measurements of both fluid loading and temperature fluctuations.

Though not apparent in the scaled data, the dependence of nanoribbon sensitivity on geometry is consistent with the theoretical prediction. The sensor geometry of 375 × 6.5 µm registered the lowest resistance changes, while the 750 × 6.5 µm, predicted to be the most sensitive, registered the largest. These results were expected, as a longer nanoribbon is exposed to more forcing and has a higher electrical resistance, while a wider nanoribbon has a higher resistance to bending and lower electrical resistance. These two properties together enable longer and thinner nanoribbons to produce greater resistance changes in response to flow. Velocity measurements using EFV will benefit from nanoribbon geometries that maximize length while minimizing cross-sectional area.

Furthermore, uncertainty in the fluid forcing remains an issue for comparison to the model. Resistance measurements...
can capture the trend due to the calibration offset of the curve, but exact values of the coefficient of drag on a flat plate perpendicular to the flow is unknown at this point in time to these authors at these low Reynolds numbers. The additional support structures that are exposed to flow also add uncertainty to the exact drag calculation, but again only affect the modeling aspect, since they are accounted for in a calibration.

From the scaling analysis of appendix A, the fundamental frequency scale, $\omega$, can be determined for the nanoribbon geometries and flows presented above. This frequency scale is found to be on the order of 100 kHz for all geometries considered in this study. This large frequency enables the sensor to react quickly to fluctuations in the flow velocity, though further work to improve the robustness and adapt the sensor for different flows is necessary.

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**Appendix A. Scaling and dynamics**

If an external loading is considered coincident with $t$, the governing equation for the deflection across the wire in time, $\tau$, is given by:

$$\rho_A \frac{\partial^2 w}{\partial \tau^2} = \left( \rho I + \frac{\rho EI}{\kappa G} \right) \frac{\partial^4 w}{\partial x^4 \partial \tau^2} + \frac{\rho I}{\kappa G} \frac{\partial^4 w}{\partial x^4 \partial \tau^4} + \frac{\rho I \eta}{\kappa AG} \frac{\partial^3 w}{\partial x^3 \partial \tau^3} + \frac{EI}{\kappa AG} \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial \tau^2} + \frac{\rho I \eta}{\kappa AG} \frac{\partial^2 w}{\partial \tau^2} + \frac{\partial^2 w}{\partial \tau^2} = -E\frac{\partial^4 w}{\partial x^4} + N\frac{\partial^2 w}{\partial x^2} + q - \frac{EI}{\kappa AG} \frac{\partial^2 q}{\partial x^2}.$$

For this wire configuration $\rho_A$ is the density of the nanowire, $A$ is the cross sectional area of the wire and $t$ is the second moment of area ($I = b^3 t/12$), $w$ is the deflection from the neutral axis, $x$ denotes the spanwise axis along the wire, $\kappa$ is the Timoshenko shear coefficient, $G$ is the shear modulus, $E$ is the elastic modulus, $\eta$ is linear damping coefficient from the surrounding fluid, $q$ is the loading per unit span and $N$ the axial tension in the wire from elastic deformation. It is assumed that the deflections are significantly smaller than the span of the wire to ensure that the deformation of the wire is elastic and

well below the yield point of common nanofilament materials (e.g. platinum). Under this assumption, $N$ can be treated as constant across the length of the wire.

Using knowledge of the geometry, fluid flow and material properties, the governing Timoshenko beam equation can be expressed in terms of nondimensional quantities and derivatives, scaled by dimensional coefficients. Scaling parameters are chosen as $w = \delta w^*$, $\tau = \omega^{-1} \tau^*$, $x = Lo^{*}$, $q = Q q^*$, where the * superscript denotes a non-dimensional scaling function. Note that $\delta$ and $\omega$ are unknown, but prescribed such that the dimensionless terms are order unity. $Q$ is the scale of the applied loading. The size of $Q$ will typically be known within a given range for a desired application. If we first assume that the $N$ is the tension arising from then net axial strain in the wire, we can scale it with $N = E A \delta^2 L o^{* 2} N^*$. Plugging these scaling relations into the governing Timoshenko equation results in:

$$\rho A \delta w^* \frac{\partial^2 w^*}{\partial \tau^2} = \frac{\rho I \delta^2}{L^2} \left[ \frac{\partial^4 w^*}{\partial x^4 \partial \tau^2} + \frac{\partial^4 w^*}{\partial x^4 \partial \tau^4} + \frac{\partial^3 w^*}{\partial x^3 \partial \tau^3} \right] + \frac{EI \delta^2}{\kappa AG L^2} \frac{\partial^2 w^*}{\partial x^2 \partial \tau^2} + \frac{\partial^2 w^*}{\partial \tau^2} = -E\frac{\partial^4 w^*}{\partial x^4} + \frac{E A \delta^2}{L^2} \left[ N \frac{\partial^2 w^*}{\partial x^2} \right] + Q q^* \frac{\partial^2 q^*}{\partial x^2 \partial \tau^2}.$$

Terms in square brackets are dimensionless and are on the order of unity and are scaled by dimensional constants.

To determine the scale of the deflection, $\delta$, the steady state balance between a uniform external loading, $q$, and the elastic deformation of the wire can be considered. All terms with derivatives in time can be neglected and the resulting governing equation is given by the following equation:

$$0 = -\frac{E b \delta^3}{Q L^4} \frac{r^2}{12 \delta^2} \left[ \frac{\partial^4 w^*}{\partial x^4} \right] + \left[ N^* \frac{\partial^2 w^*}{\partial x^2} \right] + [q^*].$$

Note that the geometric parameters for the cross sectional area and second moment of area have been plugged in.

From this form of the equation, the relative contributions of the flexural rigidity and internal stress towards balancing the external loading are revealed. In the case where the deflections are much larger than the thickness of the nanoribbon ($\delta \gg t$), the flexural rigidity provides negligible resistance to external loading, with most of the resistance coming from internal stress. In the case where all of the resistance is derived from the internal stress, a scale for $\delta$ can be determined as $\delta \sim (Q L^3 E b t^{-1})^{1/5}$. It should also be noticed that if
$Q$ is insufficiently large to achieve $\delta \gg t$, the scaling analysis reverts to the classic Euler–Bernoulli result. Using these relationships, a threshold value for $Q$ can be established to determine when the bending transitions from classic Euler–Bernoulli bending to the elongation-dominated bending where $\delta t^{-1} \gg 1 \Rightarrow Q \gg Eb(t^{-1})^2$. Furthermore, to ensure that the beam remains in the small angle approximation regime, $Q$ must be small enough to ensure that $\delta \ll L$. Using the same scaling, this criteria can be re-expressed as an upper bound on $Q$ where $Q \ll Eb(t^{-1})$.

Considering the full equation with the scale for the steady state deflection, the time scale can be determined. Assuming the local acceleration term to be leading order, the time scale can be defined as

$$\omega \sim \sqrt{\frac{Ea^2}{\rho L^4}} \sim \sqrt{\frac{Eh}{\rho L^3 qL^3}}^{1/3} \quad (A.1)$$

Plugging in the proposed time scaling and rearranging for the leading order term gives the following relation.

$$\frac{\partial^2 w^*}{\partial t^2} = \frac{\rho Da^2}{L^2} \left( \frac{1}{A L^2} + \frac{IE}{\kappa GAL^2} \right) \frac{\partial^4 w^*}{\partial x^2 \partial t^2}$$

$$+ \frac{IEb}{\kappa GAL^2} \frac{\partial^4 w^*}{\partial x^2 \partial t^2}$$

$$+ \frac{I_0 b}{\kappa A L^2} \frac{E}{\rho_0} \frac{\partial^2 w^*}{\partial t^2}$$

$$+ \frac{I_0 b}{\kappa A L^2} \frac{E}{\rho_0} \left( \frac{\partial w^*}{\partial x^2} \right) \left( \frac{\partial w^*}{\partial t^2} \right) + \frac{\eta L^2}{A \delta \rho E} \frac{\partial w^*}{\partial t^2}$$

$$= -\frac{IL^2}{\delta^2 A} \left( \frac{\partial^2 w^*}{\partial x^4} + \left[ N \frac{\partial^2 w^*}{\partial x^2} \right] + [q^*] \right)$$

$$- \frac{E L}{\kappa A L^2} \frac{\partial^2 q^*}{\partial x^2} \quad (B.1)$$

As viscous drag is the proposed method by which the fluid flow exerts a load on the nanoribbon, the scaling analysis reveals the relative importance of the various viscous terms. As $I$ is known to be very small and $G$ is assumed to be the same order as $E$, it is clear that the leading order damping term is the final term in the left hand side of the equation. Collecting the leading order terms, (as well as the Euler–Bernoulli bending term) and replacing the loading term with the fluid forcing from section 2.1, a modified damped harmonic oscillator equation can be derived to describe the deflection of the nanoribbon. If the fluid density is near the order of the beam density, one needs to account for the effect of added mass on the vibration of the beam. Including this term, the full dynamical equation can be written as:

$$(C_\rho \rho_l + \rho)_A \frac{\partial^2 w}{\partial t^2} = -E I \frac{\partial^4 w}{\partial x^4} + N \frac{\partial^2 w}{\partial x^2} + C_\rho (U - \frac{\partial w}{\partial t})$$

where $\rho_l$ is the density of the fluid and $C_\rho$ is the coefficient of added mass. Decomposing $N$ into the contribution from pre-stress and the deformation induced stress, the above equation can be re-expressed as:

$$\frac{C_\rho \rho_l + \rho_0}{\rho} \frac{\partial^2 w}{\partial t^2} = -EI \frac{\partial^4 w}{\partial x^4} + N \frac{\partial^2 w}{\partial x^2} + C_\rho (U - \frac{\partial w}{\partial t})$$

A numerical simulation of equation (A.2) was solved using a Chebyshev spectral method and a fourth order Runge–Kutta time advancement. The results were found to have good agreement with the low order deflection and strain predictions outlined in equations (8) and (9).

**Appendix B. Pre-tension and pre-deflection effects**

Closure of the nonlinear, steady-state deflection equation with a parabolic shape profile neglected the effects of finite pre-tension or pre-deflection in the unloaded nanofilament. A simple modification to equation (7) yields the new equation

$$EA\varepsilon - \sigma_0 = qL_0 \frac{L_0}{8\delta} \quad (B.1)$$

where $\sigma_0$ is the preexisting axial stress in the unloaded nanoribbon. A negative $\sigma_0$ corresponds to a pre-tension, while a positive $\sigma_0$ will be treated as an equivalent deflection of the wire in the unloaded state. Combining equations (B.1) and (6) gives a modified equation for the deflection

$$q^+ = \frac{3qL_0}{EA}$$

$$\sigma_0^+ = \frac{2\sqrt{q^+} \sigma_0}{E}$$

$$\delta = \frac{\sqrt{\delta} L_0 \left( \sigma_0^+ (q^+ + \sqrt{(q^+)^2 - (\sigma_0^+)^2})^{1/3} \right)^3}{8}$$

It is clear that when $\sigma_0 \rightarrow 0$, the relationships revert to the results in equations (8) and (9). The overall effect of both pre-tension and pre-deflection is to reduce the net change in deflection and strain experienced by the wire under load. This modified result was also validated by numerically solving equation (A.2) and modifying the axial tension term, $N$. Results from this low order model were found to have good agreement for large ranges of pre-tension, but deviate for large values of pre-deflection where the small angle approximation loses validity.

**References**


